Thermal and Momentum Diffusivity Measurements in a Turbulent Stratified Flow

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The effect of stratification on mixing coefficients is examined using measurements of velocity and temperature across the interface between a layer of warm water flowing over a pool of colder water in a channel. Velocity profiles were obtained by generating local changes in pH to produce colored tracers that are neutrally buoyant at all points in the stratified flow. Temperature profiles were measured with small thermistor beads that traverse the water regularly at selected locations. The measurements of velocity and temperature and their gradients combined with the conservation equations of mass, momentum and energy yield the transfer coefficients for heat and momentum. Parameters characterizing the functional relationships between these eddy diffusivities and the Richardson number are determined and agree well with environmental data and analytical predictions. The water channel measurements confirm that the vertical thermal diffusivity profile can be expressed in terms of a Richardson number defined by surface conditions, depth and temperature gradient as assumed for the prediction of the thermal structure in a large temperate lake. A semiempirical parameter required in the expression relating thermal diffusivity to Richardson number has been determined from the measurements.

Nomenclature

b = width of tank
c = specific heat
g = acceleration due to gravity
k = thermal conductivity
$\mathscr{K}$ = von Kármán's constant (~ 0.42)
K_H = eddy diffusivity for heat transfer, Eq. (10), (K_{H_0} neutral stability)
$K_H' = K_H - k/\rho c$
K_M = eddy diffusivity for momentum transfer, Eq. (11), (K_{M_0} neutral stability)
K_S = transfer coefficient for salt (K_{S_0} neutral stability)
p = pressure
Pr = Prandtl number
q = heat-transfer rate
R_i = gradient Richardson number [Eq. (1)]
R_{it} = Richardson number [Eq. (5)] in terms of friction velocity
Ri = gradient Richardson number [Eq. (1)]
Ri_t = Richardson number [Eq. (5)] in terms of friction velocity
R_f = flux Richardson number $Ri\, K_H/K_M$
R_{fc} = experimental constant $\simeq 0.15$
t = time
T = temperature
x = horizontal coordinate
z = vertical coordinate
ρ = density
ω_* = friction velocity $(K_M\, \partial u/\partial z)^{1/2}$
μ = viscosity
σ_1 = empirical constant [Eq. (4)]
u = horizontal velocity
w = vertical velocity
τ = shear stress
α_v = coefficient of volumetric expansion for water

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1. Introduction

VALUES for the thermal and momentum diffusivity coefficients are prime inputs which are required in the investigation of a wide variety of environmental problems. The pollution of air by smokestack effluents and of water by power-plant thermal discharges represent practical examples in which the effect of stratification on mixing coefficients plays a dominant role. In modeling studies[1] of the thermal structure of stratified lakes, recourse must be made to semiempirical representation of the thermal and momentum diffusivity coefficients as functions of the turbulence level in the surface layer, the depth, and the local temperature gradient. Uncertainty exists[2,3] concerning the exact form of the functional relationships, and the values of some parameters required are known only approximately.

In laminar homogeneous flows, heat and momentum transfer are characterized by the thermal diffusivity $k/\rho c$ and kinematic viscosity μ/ρ which are related through the properties of molecular motion by the definition of a Prandtl number, $Pr = \mu c/k \sim 6$ for water. Flows in natural bodies of water, which are almost always turbulent, may be analyzed if $k/\rho c$ and μ/ρ are replaced by appropriate eddy diffusivities for heat, K_{H_0}, and momentum, K_{M_0}, which may be several orders of magnitude larger than the laminar values. Under conditions of neutral stratification[4] $K_{H_0} \simeq K_{M_0}$, so that the Prandtl number $K_{M_0}/K_{H_0} \simeq 1$, as given by the Reynolds analogy.

When the flow is stratified, the eddy diffusivities K_H and K_M can be expressed[3] in the form $K_H = K_{H_0} f(Ri)$ and $K_M = K_{M_0} g(Ri)$ where Ri is the gradient Richardson number defined by

$$Ri = \frac{g}{\rho}\frac{\partial \rho/\partial z}{(\partial u/\partial z)^2} = -\alpha_v g \frac{\partial T/\partial z}{(\partial u/\partial z)^2} \qquad (1)$$

The Richardson number is essentially the ratio of the generation ($\partial T/\partial z > 0$) or suppression ($\partial T/\partial z < 0$) of turbulence by the buoyancy gradient to the rate at which turbulence is produced by the Reynolds' stresses. For the stable stratification (positive Ri) investigated in this paper, density increases with depth and buoyancy inhibits the transfer of heat and momentum, although not necessarily to the same degree.

As discussed in the literature,[2-9] various formulations have been advanced for the stability parameters, of which the Richardson number is one, and for predicting the effects of

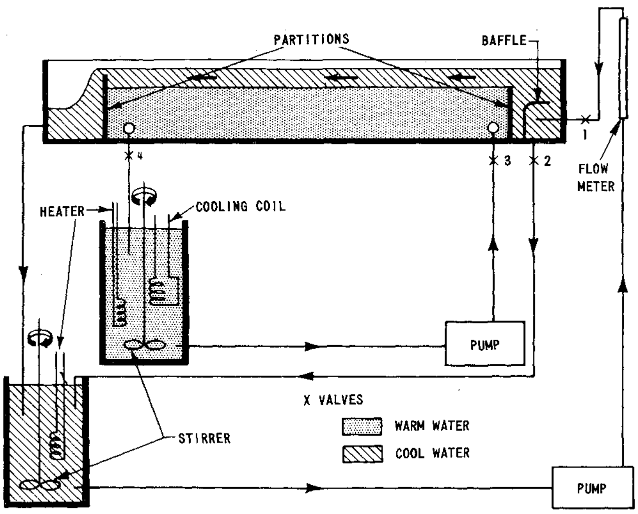

Fig. 1 Schematic arrangement of flow system.

stratification on eddy diffusivities Munk and Anderson[10] have utilized the results of several investigations[11-13] to obtain

$$K_H/K_{H_0} = (1+\tfrac{10}{3}Ri)^{-3/2} \quad (2)$$

and

$$K_M/K_{M_0} = (1+10Ri)^{-1/2} \quad (3)$$

Sundaram and Rehm[1,3,14] found good agreement between calculated temperature profiles and those measured[2] in Cayuga Lake, N.Y., by utilizing a relation of the form

$$K_H'/K_{H_0} = (1+\sigma_1 Ri_t)^{-1} \quad (4)$$

where σ_1, is a semiempirical constant, about which uncertainty exists, $K_H' = K_H - k/\rho c$ and

$$Ri_t = -\alpha_v g(z^2/\omega_*^2)\,\partial T/\partial z \quad (5)$$

In other words, the mechanical generation of turbulence is assumed to be characterized by the friction velocity at the surface $\omega_* = (K_{M_0}\partial u/\partial z)^{1/2}$ independent of the current structure below the surface.

Ellison and Turner[4] discussed the dependence of eddy viscosity on Richardson number by considering the dimensionless quantity $K_M{}^* = K_M/\omega_* z$. They showed that $K_M{}^*$ falls below its value under neutral stratification conditions, $\mathscr{K} \simeq 0.42$ (von Kármán's constant) with increasing Richardson number. Ellison[15] obtained a relation of the form

$$\frac{K_H}{K_M} = \frac{(K_{H_0}/K_{M_0})(1-R_f/R_{fc})}{(1-R_f)^2} \quad (6)$$

where the flux Richardson number is defined as $R_f = Ri\,K_H/K_M$, and R_{fc} is a constant for which Ellison and Turner[4] gave an approximate value of 0.15 based on experimental data.

The stratified-flow experiments described in this paper yield velocity and temperature profiles from which the distributions of thermal and momentum diffusivity have been derived through use of the conservation equations of mass, momentum and energy. From these results, covering a range of conditions, the dependence of the mixing coefficients on Richardson number has been compared with relationships such as Eqs. (2–6) and other data.

An important aspect of the research of wide applicability is the velocity-measuring technique which uses a tracer that is neutrally buoyant at every point in the stratified flow.

2. Experimental Techniques

A schematic arrangement of the flow system used in the experiments is given in Fig. 1. A stratified flow was produced by initiating a constant volume flow of a layer of warm water from the right-end section over a pool of colder water in the center section. The tank used was 210 cm long, 10 cm wide and 10 cm deep and the running time, which was limited by the rate at which heat penetrates downward, was typically one hour with flow velocities of 1–2 cm/sec.

As the warm water flows over the cool water, buoyancy counteracts the mixing process and a marked interface appears across which heat and momentum transfer are considerably inhibited. This is demonstrated in the temperature profiles, examples of which are shown at the top of Fig. 2. These were obtained with small thermistor beads (diameter roughly 0.3 mm) that traverse the depth of the water at selected locations at regular intervals.

The velocity profiles shown in the lower portion of Fig. 2 were produced by tracers that are neutrally buoyant at every point in the fluid. This is important because the motion of a constant-density dye would be influenced by the density gradient, giving a distorted velocity profile. The interface is visible in Fig. 2 where the warm water appears as a slightly darker region than the cold water due to differences in the index of refraction.

The velocity-measuring technique[3] represents an extension of a technique developed by Baker[16] and is based on the color change of an indicator solution when the hydrogen ion concentration (pH) is changed at a test point. The solution used consists of water containing 0.01% thymol blue. It is titrated to the end point by adding a small amount of hydrochloric acid and is initially bright orange in color. Fine tungsten wires (diameter roughly 0.1 mm) stretched vertically across the water act as cathodes, and the anode is formed by a stainless-steel wire along the tank bottom. Application of brief voltage pulses (90 v) to the two electrodes reduces the hydrogen-ion concentration at the surface of the tungsten wire corresponding to a local increase in pH. Columns of dark tracer fluid are formed at the surface of the tungsten wire and move with the fluid to yield vivid velocity profiles as shown in Fig. 2.

Three voltage pulses are applied to the electrodes. The final two pulses are timed about 27 and 30 sec after the first one. A photograph, which serves to visualize the probe wire in Fig. 2, is taken at the instant of the third pulse. The traces from the first pulse indicate the low velocities (0.04 cm/sec) near the tank bottom while those from the second pulse indicate the much higher velocities (1.2 cm/sec) near the water surface. Six equally spaced (20 cm) velocity and temperature probes were used. The experimental techniques are discussed in detail in Ref. 3.

3. Determination of the Eddy Diffusivities

To calculate the coefficients of thermal, K_H and momentum K_M, diffusivity, use is made of the equations of continuity,

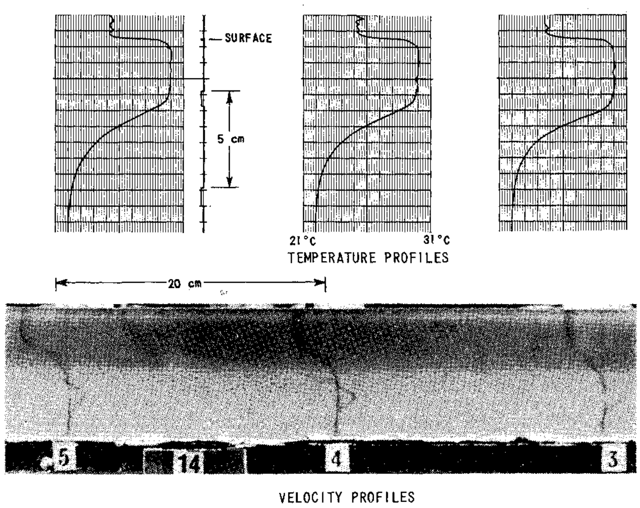

Fig. 2 Temperature and velocity profiles (run no. 14, probes 3, 4, 5).

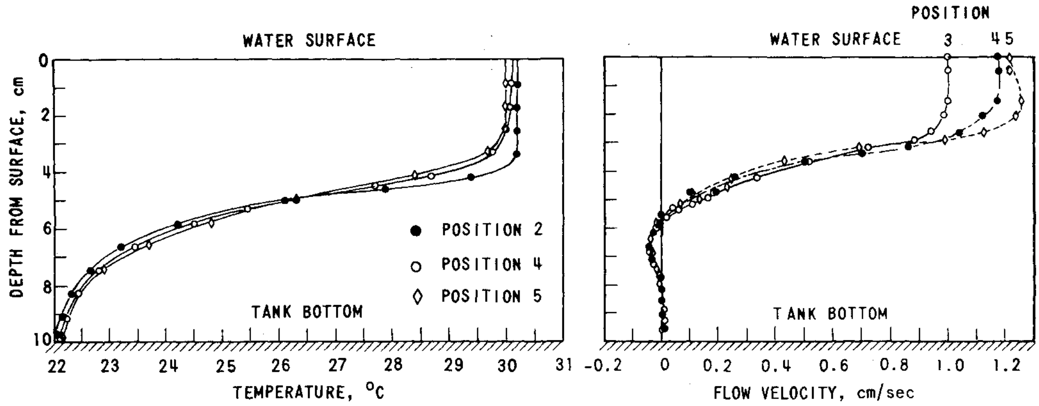

Fig. 3 Temperature, flow velocity vs depth and position, 30 min.

momentum and energy for two-dimensional flow which, for the conditions of the experiment can be expressed as

continuity

$$\partial u/\partial x + \partial w/\partial z = 0 \tag{7}$$

momentum

$$\frac{1}{\rho}\frac{\partial \tau}{\partial z} = \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} \tag{8}$$

energy

$$\frac{1}{\rho c}\frac{\partial q}{\partial z} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} \tag{9}$$

with K_H and K_M defined by

$$q/\rho c = K_H\, \partial T/\partial z \tag{10}$$

$$\tau/\rho = K_M\, \partial u/\partial z \tag{11}$$

From measurements such as those shown in Fig. 2, vertical temperature and velocity profiles were determined as functions of position and time, as indicated in Figs. 3 and 4. These measured profiles were then used to evaluate the derivatives of velocity $\partial u/\partial x, \partial u/\partial z, \partial u/\partial t$ and temperature $\partial T/\partial x, \partial T/\partial z, \partial T/\partial t$. Integration of the continuity Eq. (7) yielded the vertical velocity of w with the boundary condition that $w = 0$ at the tank bottom. The pressure gradient term $\partial p/\partial x$ was calculated as a function of depth by using the hydrostatic equation $\partial p/\partial z = -\rho g$ at different locations, x, in the tank and noting that the shear stress must be zero at the two velocity maxima observed in the low-speed (less than 0.04 cm/sec) flow near the tank bottom, Fig. 3. Integration of the momentum equation (8) from the zero shear stress point gives the shear τ as a function of depth z and Eq. (11) then yields the momentum diffusivity K_M at different depths. Similarly, integration of the energy equation (9) from the tank bottom where the thermal diffusivity, K_H, equals the laminar value (1.5×10^{-3} cm^2/sec) yields the heat-transfer rate q at different depths and Eq. (10) gives K_H as a function of depth. Details of the analysis procedure can be found in Ref. 3.

4. Discussion of Results

Diffusivity Profiles

The thermal and momentum diffusivity profiles calculated from the measured temperature and velocity profiles are presented in Fig. 5. Uncertainties in the eddy diffusivities, which were determined from measured differences of terms in the momentum and energy equations, are indicated by error bars in the figure. In the nearly constant-temperature and constant-velocity flow near the surface, the thermal diffusivity, ($K_{H_0} \simeq 0.11$ cm^2/sec) and the momentum diffusivity, ($K_{M_0} \simeq 0.15$ cm^2/sec) are approximately equal, as observed in unstratified turbulent flows.[4] Typical values in Ellison and Turner's investigation[4] at higher flow velocities are 0.25 cm^2/sec for the transfer coefficient of salt, K_S, and 0.33 cm^2/sec for K_M. At increasing depths in Fig. 5, the temperature falls, and buoyancy inhibits the transfer of heat and momentum. Both K_H and K_M decrease rapidly, although at different rates with increasing depth.

At 6 to 7 cm below the surface, a reverse flow, which is caused by the endwall, exists in the tank with a velocity of approximately 0.04 cm/sec. Both K_H and K_M have constant values in this region which are approximately 50% greater than the laminar ones so that the Prandtl number K_M/K_H has reached the laminar value, i.e., $\mu c/k \sim 6$. Near the tank bottom, the maximum flow velocity is 0.01 cm/sec and K_H and K_M are equal to their respective laminar values $k/\rho c$ and μ/ρ.

In the horizontal plane, the flow of the warm surface water can be considered roughly equivalent to a two-dimensional jet

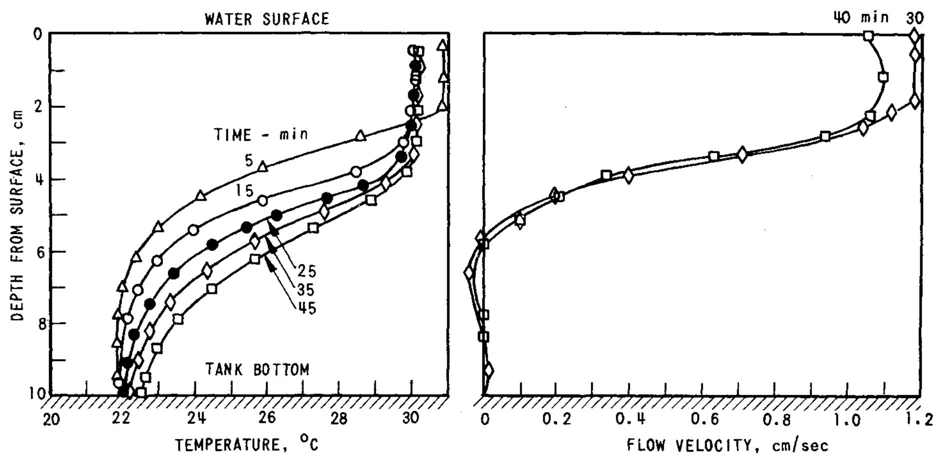

Fig. 4 Temperature, flow velocity vs depth and time, position 4.

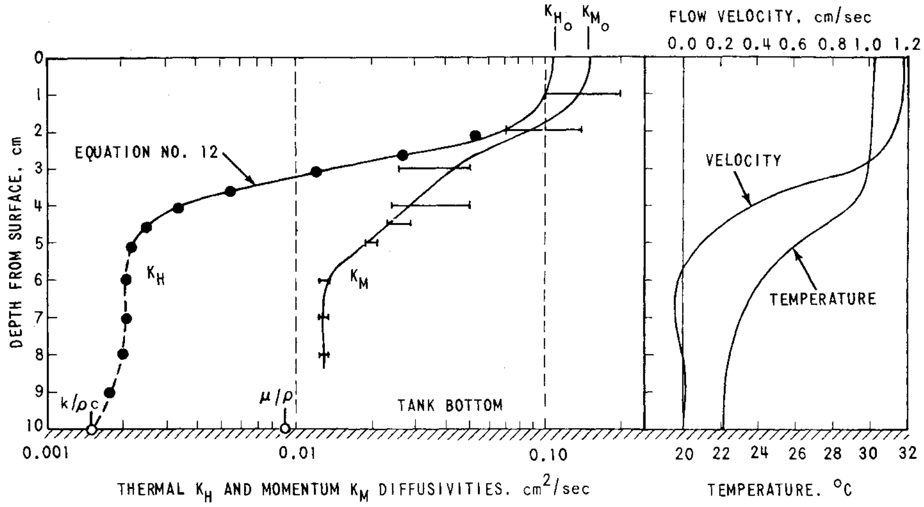

Fig. 5 Thermal and momentum diffusivity profiles.

for which Schlichting[17] gives the relation $K_{M_0} \simeq 0.01bu$ based on experimental measurements where b is the width of the jet and u the flow velocity. In the present experiments, b is approximated by the tank width (10 cm) and the flow velocity, u, equals 1.2 cm/sec so that $K_M \approx 0.12$ cm^2/sec which compares well with the observed values of $K_{M_0} = 0.15$ cm^2/sec and $K_{H_0} \approx 0.11$ cm^2/sec. In addition, combination of $K_{M_0} \approx \mathscr{K}\omega_* z$ for unstratified flows[4,5] with the definition $\omega_* = (K_{M_0} \partial u/\partial z)^{1/2}$ yields $K_{M_0} \approx \mathscr{K}^2 z^2 \partial u/\partial z$. Using this formulation and the measured velocity profile, a value for K_{M_0} of 0.175 cm^2/sec in the surface layer is obtained, so that the measured values are reasonable.

Functional Dependence of Diffusivity on Richardson Number

From the measured velocity and temperature profiles in Figs. 3–5, the gradient Richardson number, Ri, defined by Eq. (1) was calculated as a function of depth, and values between 0.2 and 26 were obtained. Using the thermal and momentum diffusivity profiles given in Fig. 5, the ratios K_M/K_{M_0}, K_H/K_{H_0} and K_M/K_H were determined and plotted against the corresponding Richardson number in Fig. 6. The results are compared in the figure with data from Jacobsen,[13] Proudman,[18] Ellison and Turner,[4] Munk and Anderson's[10] relations [Eqs. (2) and (3)] and Ellison's[15] result [Eq. (6)].

Referring to Fig. 6a, the effect of stratification on K_M/K_{M_0} is seen to be predicted quite reasonably by Eq. (3). Existing data from Jacobsen,[13] all for Richardson numbers Ri greater than 2.5, compare well with the present Calspan measurements, which extend the range in Ri down to 0.2. Similarly, as shown in Fig. 6b, Eq. (2) matches the observed variation in K_H/K_{H_0} with Richardson number within a factor of two. The Calspan data breaks away from the curve at about $K_H/K_{H_0} \approx 0.02$ since the thermal diffusivity cannot fall below the laminar value which is 0.013 for the present measurements.

Ellison and Turner[4] found scattered values for $K_M{}^* = K_M/\omega_* z$ ranging between the neutral value $\mathscr{K} = 0.42$ (von Kármán's

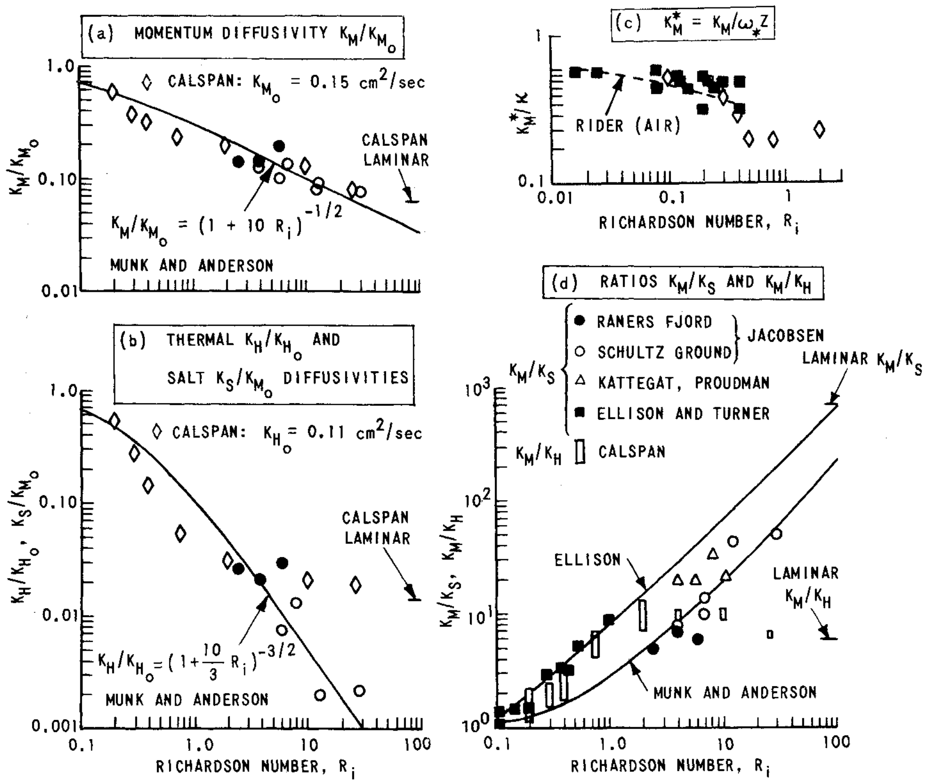

Fig. 6 Functional dependence of diffusivity on Richardson number.

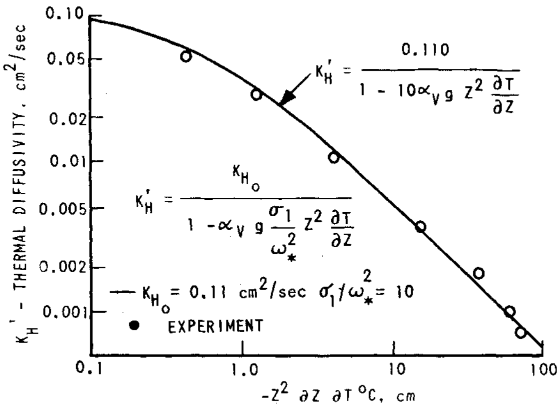

Fig. 7 Determination of parameters K_{H_0} and σ_1/ω_*^2.

constant) and 0.21 for Richardson numbers between 0 and 0.5 with an apparent tendency to decrease with increasing Richardson number as shown in Fig. 6c in which $K_M{}^*/\mathcal{K}$ is plotted against Richardson number. In the present experiments, calculated values of $K_M{}^*/\mathcal{K}$ in Fig. 6c decrease from about 1.0 in the unstratified surface layer, which helps substantiate the value of K_{M_0} observed, to roughly $\frac{1}{4}$ at $Ri = 1.0$ demonstrating the effect of stratification. Limited atmospheric data from Rider[19] shows the same trend.

Munk and Anderson[10] take the eddy diffusivities under neutral stratification conditions (K_{M_0}, K_{H_0} and K_{S_0}) to be equal and assume that for stratified flow, the turbulent transfer coefficient for salt K_S equals that for heat K_H and both differ from that for momentum K_M. These assumptions appear to be justified by the experimental results given in Fig. 6 within certain limits. In Fig. 6d, the ratios K_M/K_S (from other investigations) and K_M/K_H are compared with Munk and Anderson's[10] relation obtained by combining Eqs. (2) and (3) and Ellison's[15] result, Eq. (6). The theoretical curves, in which empirical parameters were chosen to fit various experimental results, indicate that $K_M \simeq K_S \simeq K_H$ for Richardson numbers less than 0.1. For Ri greater than about 0.3, the curves differ by a factor of three and seem to bracket the K_M/K_S data fairly well. However, stratification suppresses the turbulence in the flow and it is not unreasonable to expect that the ratio of diffusivities might approach the laminar value at high Richardson numbers. For K_M/K_S this is roughly 700 compared with a value of 6 for K_M/K_H.

The available measurements for K_M/K_S given in Fig. 6d show a continuous rise towards the laminar value following the analytical curves. In contrast, the present data for K_M/K_H reach a plateau of about 10 at Ri between 1 and 10 and then drop towards the laminar value of 6. This behavior may be due to the low turbulence level of the present experiments, and higher values for K_M/K_H may be achieved in other environments. However, it is possible that K_M/K_H will not increase continuously with Richardson number as K_M/K_S does, and consequently caution should be exercised in equating values of K_M/K_S and K_M/K_H at high Richardson numbers.

Thermal Diffusivity from Surface Conditions

In a theoretical analysis of thermal pollution Sundaram and Rehm[14] express the effect of buoyancy on the eddy diffusivity for heat in the form,

$$K_H'/K_{H_0} = [1 - \alpha_v g(\sigma_1/\omega_*^2) z^2\, \partial T/\partial z]^{-1} \qquad (12)$$

which can be obtained by combining Eqs. (4) and (5). σ_1 is a semiempirical parameter and $\omega_* = (K_{M_0}\, \partial u/\partial z)^{1/2}$ is the friction velocity at the surface. The present experiment is suitable for the determination of σ_1.

In Fig. 7, the experimental values of K_H' are plotted against the corresponding measurements of $z^2\, \partial T/\partial z$. By fitting a curve to the experimental K_H' distribution, values of 0.11 were established for K_{H_0}, and 10 for σ_1/ω_*^2. From the measurements (Fig. 5), the friction velocity ω_* was calculated to be roughly 0.1 cm/sec which yields a value for σ_1 of approximately 0.1. In Fig. 5, Eq. (12), with $\sigma_1 = 0.1$ chosen to fit the data, gives very good agreement with the measured distribution of thermal diffusivity.

Sundaram and Rehm[3,14] found that $\sigma_1 = 0.1$ gave good agreement between theoretical and measured temperature profiles for Cayuga Lake. The exact agreement between the two values of σ_1 is considered fortuitous and significant only in that the correct order of magnitude has been established for stratified flows of water. Uncertainties[3] in σ_1 range from a theoretical value of 0.08 to atmospheric values of 1.1. The value determined from the measurements is probably accurate within a factor of two.

Values of K_{H_0} obtained in flow environments ranging from the present experiments in a small water tank to measurements in Cayuga Lake[2] and the Ocean,[10] as presented in Fig. 8, show the effect of the local flow velocity on K_{H_0} over a span of six orders of magnitude. The higher values of K_{H_0} observed in the ocean reflect the increased scale since $K_{H_0} \approx \mathcal{K}\omega_* z$. The eddy diffusivity, K_{M_0}, for momentum transfer under neutral stability conditions shows the same trend since $K_{M_0} \approx K_{H_0}$.

5. Summary

A description has been given of the successful application of a technique[16,3] for the measurement of velocity in a stratified flow using a neutrally-buoyant tracer that is unaffected by the density gradient. The tracer is generated by brief applications of a voltage pulse to fine tungsten wires in a water solution of 0.01% thymol blue titrated to the end point. The resulting change in hydrogen ion concentration increases the pH of the flow at the surface of the wires producing a local color change which traces the subsequent motion of the fluid. The value of this technique for the investigation of a wide range of stratified flow problems is clear because the motion of a constant-density dye used in standard dye-visualization experiments can be influenced by the density gradient, and hence give a distorted velocity profile.

The velocity-measuring technique along with small thermistor beads that traverse the tank were used to measure vertical velocity and temperature profiles in the flow of a layer of warm water over a pool of colder water. These profiles combined with the conservation equations of mass, momentum and energy yielded the transfer coefficients of heat K_H and momentum K_M across the interface between the warm and cold water.

The effect of stable stratification on the vertical profiles of K_H and K_M was examined by determining the reduction in

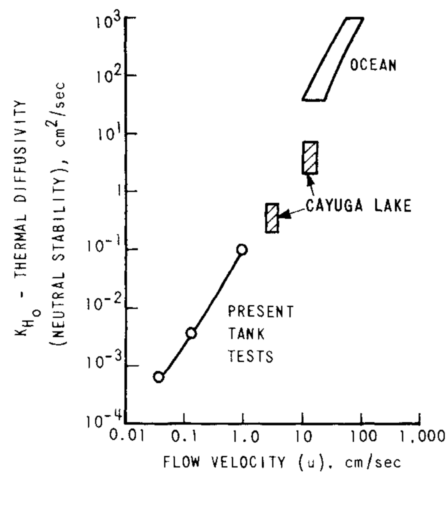

Fig. 8 Thermal diffusivity (neutral stability) vs flow velocity.

K_H and K_M below their neutral-stability values K_{H_0} and K_{M_0} in the unstratified surface layer as a function of the gradient Richardson number, Ri. The relations of Munk and Anderson,[10] based on environmental data[13,18] give reasonable estimates of the observed variation in K_H/K_{H_0} and K_M/K_{M_0} with Ri. The parameter $K_M/\omega_* z$, which equals roughly 0.42 (von Kármán's constant, $\mathcal{K}$) in unstratified flows is shown to decrease with increasing Richardson number to about 0.10 at an Ri of 1.0, in general agreement with the results of other investigations.[4,19]

Ellison[15] and Munk and Anderson[10] develop equations to predict K_M/K_S as a function of Richardson number where K_S, the transfer coefficient of salt, is taken equal to K_H. At Richardson numbers lower than about 0.1 $K_M/K_S = K_M/K_H \simeq 1.0$, and $K_{M_0} \simeq K_{H_0} \simeq K_{S_0}$ in turbulent unstratified flows. For Ri between 0.1 and 2.0 Ellison's and Munk and Anderson's equations which differ by up to a factor of three, seem to bracket the measurements. For Ri greater than two, the predictive equations, developed from measurements of K_M/K_S, continue to increase towards the laminar value $K_M/K_S \simeq 700$. In contrast, the present measurements of K_M/K_H level off at a value of approximately 10 at Ri between one and ten and then fall toward the laminar value of six. This behavior, which may be partially a result of the low turbulence level, indicates that care should be exercised in equating values of K_M/K_S and K_M/K_H at high Richardson numbers.

Sundaram and Rehm[3] obtain good agreement between the measured and predicted temperature profiles in a large temperate lake by expressing the vertical distribution of thermal diffusivity in terms of the friction velocity at the surface, the depth, and local temperature gradient. In their analysis, a semiempirical parameter σ_1 in the expression $K_H'/K_{H_0} = (1+\sigma_1 Ri_t)^{-1}$ was taken to be equal to 0.1. In the present water-channel experiments, a value for σ_1 was determined from the measurements and it was observed that $\sigma_1 = 0.1$ gave an excellent fit to the measured profile of thermal diffusivity.

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